Time Series

EXERCISE 4.1 [PAGES 66 - 67]

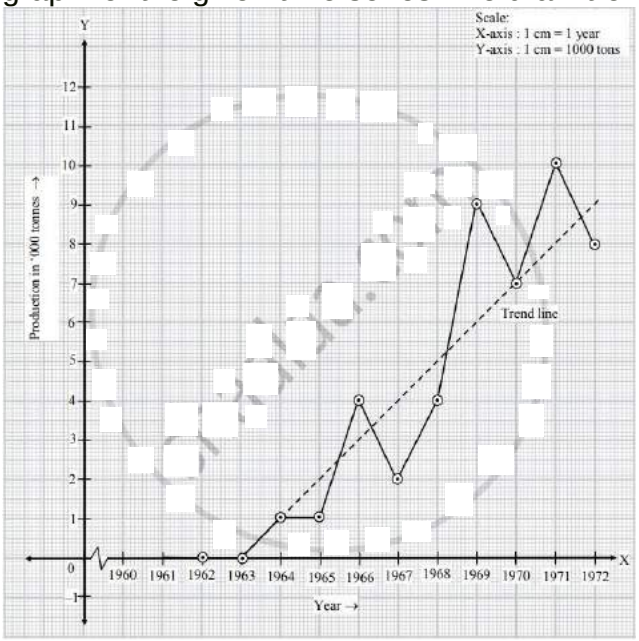
Exercise 4.1 | Q 1 | Page 66

The following data gives the production of bleaching powder (in '000 tons) for the years 1962 to 1972.

Year	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972
Production	0	0	1	1	4	2	4	9	7	10	8

Fit a trend line by graphical method to the above data.

Solution: Taking year on X-axis and production on Y-axis, we plot the points for production corresponding to the years. Joining the points by straight lines, we get the graph for the given time series. We draw trend line as shown in the figure.



Exercise 4.1 | Q 2 | Page 66

Use the method of least squares to fit a trend line to the data in Problem 1 above. Also, obtain the trend value for the year 1975.

Solution: In the given problem, n = 11 (odd), middle t- values is 1967, h = 1

$$u = \frac{\text{t - middle value}}{h} = \frac{t - 1967}{1} = t - 1967$$

We obtain the following table.

Year (t)	Production(y_t) (in '000 tonnes)	u = t 1967	u^2	uy_t	Trend value
1962	0	–5	25	0	–1.0002
1963	0	–4	16	0	0.0362
1964	1	–3	9	–3	1.0726
1965	1	–2	4	–2	2.1090
1966	4	–1	1	–4	3.1454
1967	2	0	0	0	4.1818
1968	4	1	1	4	5.2182
1969	9	2	4	18	6.2546
1970	7	3	9	21	7.2910
1971	10	4	16	40	8.3274
1972	8	5	25	40	9.3638
Total	**46**	**0**	**110**	**114**	

From the table, n = 11,

$$\sum y_t = 46, \sum u = 0, \sum u^2 = 110, \sum uy_t = 114$$

The two normal equations are:

$$\sum y_t = na' + b' \sum u \text{ and } \sum uy_t, = a' \sum u + b' \sum u^2$$

$\therefore$ 46 = 11a' + b'(0) ...(i) and

$$114 = a' = \frac{46}{11} = 4.1818$$

From (ii), $b' = \frac{114}{110} = 1.0364$

∴ The equation of the trend line is $y_t = a' + b'u$

i.e., $y_t = 4.1818 + 1.0364\ u$, where $u = t - 1967$

Now, for $t = 1975$, $u = 1975 - 1967 = 8$

∴ $y_t = 4.1818 + 1.0364 \times 8 = 12.473$.

Exercise 4.1 | Q 3 | Page 66

Obtain the trend line for the above data using 5 yearly moving averages.

Solution: Construct the following table to obtain 5 yearly moving averages in data of problem 1.

Year T	Production y_t (in' 000 tonnes)	5 – yearly moving total	5 – yearly moving averages trend value
1962	0	–	–
1963	0	–	–
1964	1	6	1.2
1965	1	8	1.6
1966	4	12	2.4
1967	2	20	4
1968	4	26	5.2
1969	9	32	6.4
1970	7	38	7.6
1971	10	–	–
1972	8	–	–

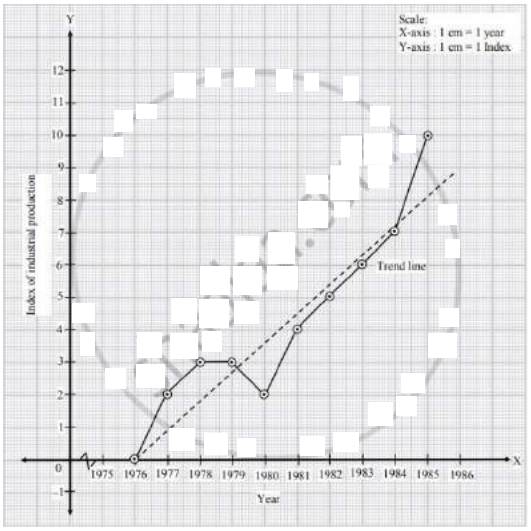
Exercise 4.1 | Q 4 | Page 66

The following table shows the index of industrial production for the period from 1976 to 1985, using the year 1976 as the base year.

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985
Index	0	2	3	3	2	4	5	6	7	10

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and index on Y-axis. We plot the points for indices corresponding to the years. Joining these points we get the graph of the given time series. We draw trend line as shown in the figure.



Exercise 4.1 | Q 5 | Page 66

Fit a trend line to the data in Problem 4 above by the method of least squares. Also, obtain the trend value for the index of industrial production for the year 1987.

Solution: In the given problem, x = 10(even), two middle t – values are 1980 and 1981, h = 1

$$u = \frac{t - \text{mean of two middle values}}{\frac{h}{2}} = \frac{t - 1980.5}{\frac{1}{2}} = 2(t - 1980.5)$$

We obtain the following table.

Year (t)	Index of industrial production y_t	u = 2 (t - 1980.5)	u^2	uy_t	Trend value
1976	0	–9	81	0	0.1635
1977	2	–7	49	–14	1.0605
1978	3	–5	25	–15	1.9575
1979	3	–3	9	–9	2.8545
1980	2	–1	1	–2	3.7515
1981	4	1	1	4	4.6485
1982	5	3	9	15	5.5455
1983	6	5	25	30	6.4425
1984	7	7	49	49	7.3395

1985	10	9	81	90	8.2365
Total	**42**	**0**	**330**	**148**	

From the table, n = 10,

$$\sum y_t = 42, \sum u = 0, \sum u^2 = 330, \sum uy_t = 148$$

The two normal equations are :

$$\sum y_t = \text{na}' + \text{b}' \sum u \text{ and } \sum uy_t = \text{a}' \sum u + \text{b}' \sum u^2$$

$\therefore$ 42 = 10a' + b'(0) ...(i) and

148 = a'(0) + b'(330) ...(ii)

From (i), a' = $\frac{42}{10}$ = 4.2

From (ii), b' = $\frac{148}{330}$ = 0.4485

$\therefore$ The equation of the trend line is y_t = a' + b'u

i.e., y_t = 4.2 + 0.4485 u, where u = 2(t – 1980.5)

$\therefore$ Now, For t = 1987, u = 2(1987 – 1980.5) = 2 x 6.5 = 13

$\therefore$ y_t = 4.2 + 0.4485 x 13 = 10.0305.

Exercise 4.1 | Q 6 | Page 66

Obtain the trend values for the data in problem 4 using 4-yearly centered moving averages.

Solution: Construct the following table for finding 4 yearly centered moving average for the data in problem 4.

Year t	Index y_t	4–yearly moving total	4–yearly moving averages	2 unit moving total	4 yearly centred moving average (trend values)

1976	0				
1977	2				
		8	2		
1978	3			4.5	2.5
		10	2.5		
1979	3			5.5	2.75
		12	3		
1980	2			6.5	3.25
		14	3.5		
1981	4			7.75	3.875
		17	4.25		
1982	5			9.75	4.875
		22	5.5		
1983	6			12.5	6.25
		28	7		
1984	7				
1985	10				

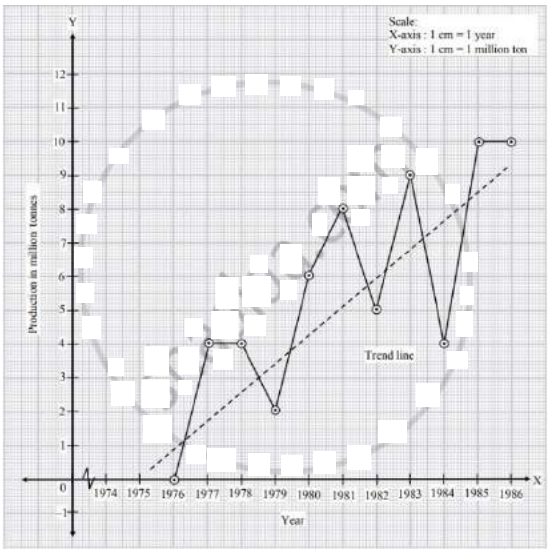
Exercise 4.1 | Q 7 | Page 66

The following table gives the production of steel (in millions of tonnes) for years 1976 to 1986.

Year	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Production	0	4	4	2	6	8	5	9	4	10	10

Fit a trend line to the above data by the graphical method.

Solution: Taking year on X-axis and production on Y-axis, we plot the points for production corresponding to years. Joining these points by straight lines, we get the graph of the given time series. We draw trend line as shown in the figure.



Exercise 4.1 | Q 8 | Page 66

Fit a trend line to the data in Problem 7 by the method of least squares. Also, obtain the trend value for the year 1990.

Solution: In the given problem, n = 11 (odd), middle t- values is 1981, h = 1

$$u = \frac{t - \text{middle value}}{h} = \frac{t - 1981}{1} = t - 1981$$

We obtain the following table.

Year t	Production y_t	u = t–1981	u^2	uy_t	Trend Value
1976	0	–5	25	0	1.6819
1977	4	–4	16	–16	2.4728
1978	4	–3	9	–12	3.2637
1979	2	–2	4	–4	4.0546
1980	6	–1	1	–6	4.8455
1981	8	0	0	0	5.6364
1982	5	1	1	5	6.4273
1983	9	2	4	18	7.2182
1984	4	3	9	12	8.0091
1985	10	4	16	40	8.8
1986	10	5	25	50	9.5909
Total	**62**	**0**	**110**	**87**	

From the table, n = 11,

$$\sum y_t = 62, \sum u = 0, \sum u^2 = 110, \sum uy_t = 87$$

The two normal equations are :

$$\sum y_t = \text{na}\prime + \text{b}\prime \sum u \text{ and } \sum uy_t = \text{a}\prime \sum u + \text{b}\prime \sum u^2$$

$\therefore$ 62 = 11a' + b'(0) ...(i) and

87 = a'(0) + b'(110) ...(ii)

From (i), $a' = \frac{62}{11} = 5.6364$

From (ii), $b' = \frac{87}{110} = 0.7909$

$\therefore$ The equation of the trend line is $y_t = a' + b'u$

i.e., $y_t = 5.6364 + 0.7909\ u$, where $u = t - 1981$

$\therefore$ Now, For t = 1990, u = 1990 – 1981= 9

$\therefore$ $y_t = 5.6364 + 0.7909 \times 9 = 12.7545$.

Exercise 4.1 | Q 9 | Page 66

Obtain the trend values for the above data using 3-yearly moving averages.

Solution: Construct the following table for obtaining 3-yearly moving averages for the data in problem 7.

Year t	Production y_t	3–yearly moving total	3–yearly moving averages trend value
1976	0	–	–
1977	4	8	2.6667
1978	4	10	3.3333
1979	2	12	4
1980	6	16	5.3333

1981	8	19	6.3333
1982	5	22	7.3333
1983	9	18	6
1984	4	23	7.6667
1985	10	24	8
1986	10	–	–

Exercise 4.1 | Q 10 | Page 67

The following table shows the production of gasoline in U.S.A. for the years 1962 to 1976.

Year	1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
Production (Million Barrels)	10	0	1	1	2	3	4	5	6	7	8	9	8	9	10

i. Obtain trend values for the above data using 5-yearly moving averages.

ii. Plot the original time series and trend values obtained above on the same graph.

Solution: i.

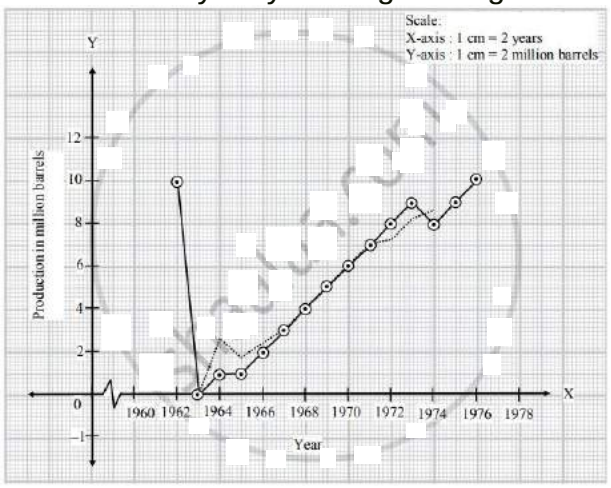
Year t	**Production (millions of barrels) y_t**	**5–yearly moving total**	**5–yearly moving averages trend value**
192	10	–	–
1963	0	–	–
1964	1	14	2.8
1965	1	7	1.4
1966	2	11	2.2
1967	3	15	3
1968	4	20	4
1969	5	25	5
1970	6	30	6

1971	7	35	7
1972	8	38	7.6
1973	9	41	8.2
1974	8	44	8.8
1975	9	–	–
1976	10	–	–

ii.

Taking year on X-axis and production trend on Y-axis, we plot the points for production corresponding to years to get the graph of time series and plot the points for trend values corresponding to years to get the graph of trend as shown in the adjoining figure.

Production: 5 yearly moving Average: ----------



MISCELLANEOUS EXERCISE 4 [PAGES 67 - 70]

Miscellaneous Exercise 4 | Q 1.01 | Page 67

Choose the correct alternative :

Which of the following can't be a component of a time series?

1. Seasonality
2. Cyclical
3. Trend
4. **Mean**

Solution: Mean**.**

Miscellaneous Exercise 4 | Q 1.02 | Page 68

Choose the correct alternative :

The first step in time series analysis is to _____.

1. perform regression calculations
2. calculate a moving average
3. **plot the data on a graph**
4. identify seasonal variation

Solution: The first step in time series analysis is to **plot the data on a graph**.

Miscellaneous Exercise 4 | Q 1.03 | Page 68

Choose the correct alternative :

Time-series analysis is based on the assumption that _____.

1. random error terms are normally distributed.
2. the variable to be forecast and other independent variables are correlated.
3. **past patterns in the variable to be forecast will continue unchanged into the future.**
4. the data do not exhibit a trend.

Solution: Time-series analysis is based on the assumption that **past patterns in the variable to be forecast will continue unchanged into the future**.

Miscellaneous Exercise 4 | Q 1.04 | Page 68

Choose the correct alternative :

Moving averages are useful in identifying _____.

1. seasonal component
2. irregular component
3. **trend component**
4. cyclical component

Solution: Moving averages are useful in identifying **trend component**.

Miscellaneous Exercise 4 | Q 1.05 | Page 68

Choose the correct alternative :

We can use regression line for past data to forecast future data. We then use the line which_____.

1. **minimizes the sum of squared deviations of past data from the line**
2. minimizes the sum of deviations of past data from the line
3. maximizes the sum of squared deviations of past data from the line

4. maximizes the sum of deviations of past data from the line

Solution: We can use regression line for past data to forecast future data. We then use the line which **minimizes the sum of squared deviations of past data from the line**.

Miscellaneous Exercise 4 | Q 1.06 | Page 68

Choose the correct alternative :

Which of the following is a major problem for forecasting, especially when using the method of least squares?

1. The past cannot be known.
2. The future is not entirely certain.
3. The future exactly follows the patterns of the past.
4. **The future may not follow the patterns of the past.**

Solution: The future may not follow the patterns of the past**.**

Miscellaneous Exercise 4 | Q 1.07 | Page 68

Choose the correct alternative :

An overall upward or downward pattern in an annual time series would be contained in which component of the times series?

1. **Trend**
2. Cyclical
3. Irregular
4. Seasonal

Solution: Trend**.**

Miscellaneous Exercise 4 | Q 1.08 | Page 68

Choose the correct alternative :

The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base or zero year. Y = 500 + 60X (in 1000 Rs). The estimated sales for 1984 (in 1000 Rs) is:

1. **Rs 500**
2. Rs 560
3. Rs 1,040
4. Rs 1,100

Solution: According to the given information

For year 1984, X = 0

∴ Y = 500 + 60(0) = **500**.

Miscellaneous Exercise 4 | Q 1.09 | Page 68

Choose the correct alternative :

What is a disadvantage of the graphical method of determining a trend line?

1. Provides quick approximations
2. **Is subject to human error**
3. Provides accurate forecasts
4. Is too difficult to calculate

Solution: Is subject to human error**.**

Miscellaneous Exercise 4 | Q 1.1 | Page 68

Choose the correct alternative :

Which component of time series refers to erratic time series movements that follow no recognizable or regular pattern?

1. Trend
2. Seasonal
3. Cyclical
4. **Irregular**

Solution: Irregular.

Miscellaneous Exercise 4 | Q 2.01 | Page 68

Fill in the blank :

_____ component of time series is indicated by a smooth line.

Solution: **trend** component of time series is indicated by a smooth line.

Miscellaneous Exercise 4 | Q 2.02 | Page 68

Fill in the blank :

_____ component of time series is indicated by periodic variation year after year.

Solution: **seasonal** component of time series is indicated by periodic variation year after year.

Miscellaneous Exercise 4 | Q 2.03 | Page 68

Fill in the blank :

_____ component of time series is indicated by a long wave spanning two or more years.

Solution: **cyclical** component of time series is indicated by a long wave spanning two or more years.

Miscellaneous Exercise 4 | Q 2.04 | Page 68

Fill in the blank :

_____ component of time series is indicated by up and down movements without any pattern.

Solution: **irregular** component of time series is indicated by up and down movements without any pattern.

Miscellaneous Exercise 4 | Q 2.05 | Page 68

Fill in the blank :

Addictive models of time series _____ independence of its components.

Solution: Addictive models of time series **assume** independence of its components.

Miscellaneous Exercise 4 | Q 2.06 | Page 68

Fill in the blank :

Multiplicative models of time series _____ independence of its components.

Solution: Multiplicative models of time series **does not assume** independence of its components.

Miscellaneous Exercise 4 | Q 2.07 | Page 68

Fill in the blank :

The simplest method of measuring trend of time series is _____

Solution: The simplest method of measuring trend of time series is **graphical**.

Miscellaneous Exercise 4 | Q 2.08 | Page 68

Fill in the blank :

The method of measuring trend of time series using only averages is _____

Solution: The method of measuring trend of time series using only averages is **moving average**.

Miscellaneous Exercise 4 | Q 2.09 | Page 69

Fill in the blank :

The complicated but efficient method of measuring trend of time series is _____.

Solution: The complicated but efficient method of measuring trend of time series is **least square**.

Miscellaneous Exercise 4 | Q 2.1 | Page 69

Fill in the blank :

The graph of time series clearly shows _____of it is monotone.

Solution: The graph of time series clearly shows **Trend** of it is monotone.

Miscellaneous Exercise 4 | Q 3.01 | Page 69

State whether the following is True or False :

The secular trend component of time series represents irregular variations.

1. True
2. **False**

Solution: The secular trend component of time series represents irregular variations **False**.

Miscellaneous Exercise 4 | Q 3.02 | Page 69

State whether the following is True or False :

Seasonal variation can be observed over several years.

1. **True**
2. False

Solution: Seasonal variation can be observed over several years **True**.

Miscellaneous Exercise 4 | Q 3.03 | Page 69

State whether the following is True or False :

Cyclical variation can occur several times in a year.

1. True
2. **False**

Solution: Cyclical variation can occur several times in a year **False**.

Miscellaneous Exercise 4 | Q 3.04 | Page 69

State whether the following is True or False :

Irregular variation is not a random component of time series.

1. True
2. **False**

Solution: Irregular variation is not a random component of time series **False**.

Miscellaneous Exercise 4 | Q 3.05 | Page 69

State whether the following is True or False :

Additive model of time series does not require the assumption of independence of its components.

1. True
2. **False**

Solution: Additive model of time series does not require the assumption of independence of its components **False**.

Miscellaneous Exercise 4 | Q 3.06 | Page 69

State whether the following is True or False :

Multiplicative model of time series does not require the assumption of independence of its components.

1. **True**
2. False

Solution: Multiplicative model of time series does not require the assumption of independence of its components **True**.

Miscellaneous Exercise 4 | Q 3.07 | Page 69

State whether the following is True or False :

Graphical method of finding trend is very complicated and involves several calculations.

1. True

2. **False**

Solution: Graphical method of finding trend is very complicated and involves several calculations **False**.

Miscellaneous Exercise 4 | Q 3.08 | Page 69

State whether the following is True or False :

Moving average method of finding trend is very complicated and involves several calculations.

1. True
2. **False**

Solution: Moving average method of finding trend is very complicated and involves several calculations **False**.

Miscellaneous Exercise 4 | Q 3.09 | Page 69

State whether the following is True or False :

Least squares method of finding trend is very simple and does not involve any calculations.

1. True
2. **False**

Solution: Least squares method of finding trend is very simple and does not involve any calculations **False**.

Miscellaneous Exercise 4 | Q 3.1 | Page 69

State whether the following is True or False :

All the three methods of measuring trend will always give the same results.

1. True
2. **False**

Solution: All the three methods of measuring trend will always give the same results **False**.

Miscellaneous Exercise 4 | Q 4.01 | Page 69

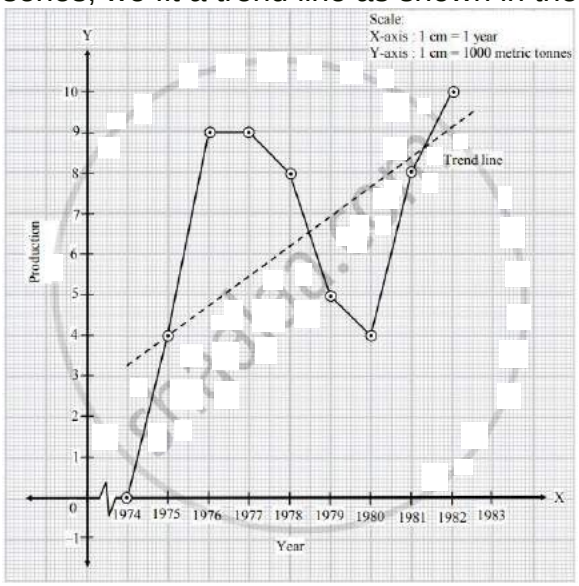
Solve the following problem :

The following table shows the production of pig-iron and ferro- alloys (‘000 metric tonnes)

Year	1974	1975	1976	1977	1978	1979	1980	1981	1982
Production	0	4	9	9	8	5	4	8	10

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and production on Y-axis, we plot the points for production corresponding to years. Joining these points, we get the graph of time series, we fit a trend line as shown in the figure.



Miscellaneous Exercise 4 | Q 4.02 | Page 70

Solve the following problem :

Obtain trend values for data in Problem 19 using 3-yearly moving averages.

Solution: Construct the following table for finding 3-yearly moving averages:

Year t	**Yield (in '000 tonnes) y_t**	**3–yearly moving total**	**3–yearly moving averages trend value**
1959	0	–	–
1960	1	3	1
1961	2	6	2
1962	3	6	2
1963	1	4	1.3333

1964	0	5	1.6667
1965	4	5	1.6667
1966	1	7	2.3333
1967	2	13	4.3333
1968	10	–	–

Miscellaneous Exercise 4 | Q 4.03 | Page 69

Solve the following problem :

Obtain trend values for data in Problem IV (1) using 5-yearly moving averages.

Solution: Construct the following table for obtaining 5-yearly moving average for the data in problem 1:

Year t	Production	5-yearly moving total	5-yearly moving averages trend value
1974	0	–	–
1975	4	–	–
1976	9	30	6
1977	9	35	7
1978	8	35	7
1979	5	34	6.8
1980	4	35	7
1981	8	–	–
1982	10	–	–

Miscellaneous Exercise 4 | Q 4.04 | Page 69

Solve the following problem :

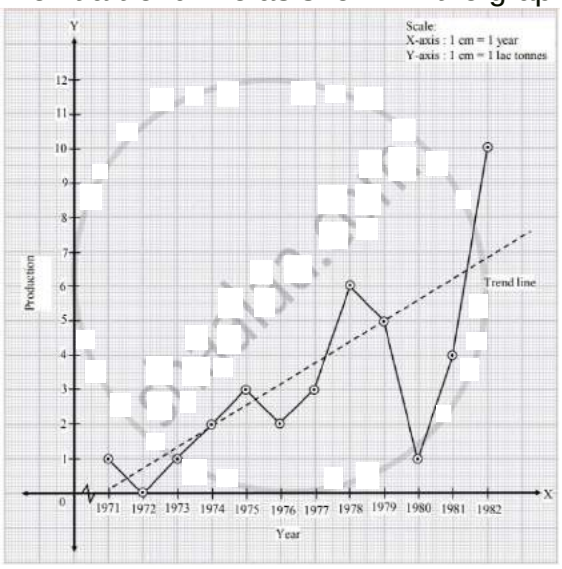
Following table shows the amount of sugar production (in lac tonnes) for the years 1971 to 1982.

Year	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982
Production	1	0	1	2	3	2	3	6	5	1	4	10

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and production on Y-axis, we plot the points for production corresponding to years. Joining these points we get the graph of time series.

We fit a trend line as shown in the graph.



Miscellaneous Exercise 4 | Q 4.05 | Page 69

Solve the following problem :

Fit a trend line to data in Problem 4 by the method of least squares.

Solution: In the given problem, n = 12 (odd), middle t – value is 1976, h = 1

$$u = \frac{t - \text{middle value}}{\frac{h}{2}} = \frac{t - 1976.5}{\frac{1}{2}} = 2(t - 1976.5)$$

We obtain the following table.

Year t	Production y_t	u = 2(t – 1976.5)	u^2	uy_t	Trend Value
1971	1	–11	121	–11	0.0900
1972	0	–9	81	0	0.6494
1973	1	–7	49	–7	1.2088
1974	2	–5	25	–10	1.7682
1975	3	–	9	–9	2.3276
1976	2	–1	1	–2	2.8870
1977	3	1	1	3	3.4464
1978	6	3	9	18	4.0058
1979	5	5	25	25	4.5652
1980	1	7	49	7	5.1246
1981	4	9	81	36	5.6840
1982	10	11	121	110	6.243

Total	38	0	572	160	

From the table, n = 12,

$$\sum y_t = 38, \sum u = 0, \sum u^2 = 572, \sum uy_t = 160$$

The two normal equations are:

$$\sum y_t = \text{na}\prime + \text{b}\prime \sum u \text{ and } \sum uy_t, = a\prime \sum u + b\prime \sum u^2$$

$\therefore$ 38 = 12a' + b'(0) ...(i) and

160 = a'(0) + b'(572) ...(ii)

From (i), a' = $\frac{38}{12}$ = 3.1667

From (ii), b' = $\frac{160}{572}$ = 0.2797

$\therefore$ The equation of the trend line is y_t = a' + b'u

i.e., y_t = 3.1667 + 0.2797 u, where u = 2(t – 1976.5).

Miscellaneous Exercise 4 | Q 4.06 | Page 69

Solve the following problem :

Obtain trend values for data in Problem 4 using 4-yearly centered moving averages.

Solution: Construct the following table for obtaining 4-yearly centered moving average for the data in problem 4.

Year t	Production y_t	4–yearly moving total	4–yearly moving average	2 unit moving total	4–yearly centred moving averages trend value
1971	1				
1972	0				
		4	1		
1973	1			2.5	1.25

		6	1.5		
1974	2			3.5	1.75
		8	2		
1975	3			4.5	2.25
		10	2.5		
1976	2			6	3
		14	3.5		
1977	3			7.5	3.75
		16	4		
1978	6			7.75	3.875
		15	.75		
1979	5			7.75	3.875
		16	4		
1980	1			9	4.5
		20	5		
1981	4				
1982	10				

Miscellaneous Exercise 4 | Q 4.07 | Page 69

Solve the following problem :

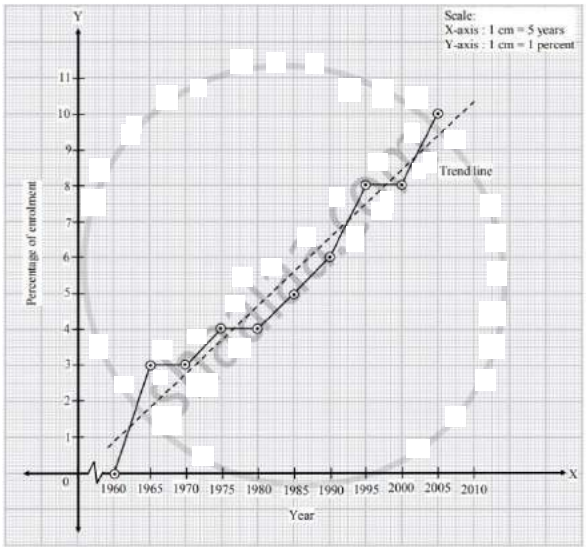
The percentage of girls' enrollment in total enrollment for years 1960-2005 is shown in the following table.

Year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
Percentage	0	3	3	4	4	5	6	8	8	10

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and percentage of enrolment on Y-axis, we plot the points for enrolment corresponding to years. Joining these points, we get the graph of

time series. We fit the trend line as shown in the following graph.



Miscellaneous Exercise 4 | Q 4.08 | Page 69

Solve the following problem :

Fit a trend line to the data in Problem 7 by the method of least squares.

Solution: In the given problem, n = 10 even), middle t – value is 1980 and 1985, h = 5

$$u = \frac{t - \text{mean of two middle values}}{\frac{h}{2}} = \frac{t - 1985.5}{\frac{5}{2}} = \frac{2(t - 1985.5)}{5}$$

We obtain the following table.

Year t	Percentage of enrolment y_t	$u = \frac{2(t-1982.5)}{5}$	u^2	uy_t	Trend Value
1960	0	–9	81	0	0.8187
1965	3	–7	49	–21	1.7701
1970	3	–5	25	–15	2.7215
1975	4	–3	9	–12	3.6729
1980	4	–1	1	–4	4.6243
1985	5	1	1	5	5.5757
1990	6	3	9	18	6.5271
1995	8	5	25	40	7.4785
2000	8	7	49	56	8.4299
2005	10	9	81	90	9.3813
Total	**51**	**0**	**330**	**157**	

From the table, n = 10,

$$\sum y_t = 51, \sum u = 0, \sum u^2 = 330, \sum uy_t = 157$$

The two normal equations are:

$$\sum y_t = \text{na}\prime + \text{b}\prime \sum u \text{ and } \sum uy_t, = a\prime \sum u + b\prime \sum u^2$$

$\therefore$ 51 = 10a' + b'(0) ...(i) and

157 = a'(0) + b'(330) ...(ii)

From (i), $a' = \frac{51}{10} = 5.1$

From (ii), $b' = \frac{157}{330} = 0.4757$

$\therefore$ The equation of the trend line is $y_t = a' + b'u$

i.e., $y_t = 5.1 + 0.4757\ u$, where $u = \frac{2(t-1985.5)}{5}$.

Miscellaneous Exercise 4 | Q 4.09 | Page 69

Solve the following problem :

Obtain trend values for the data in Problem 7 using 4-yearly moving averages.

Solution: Construct the following table for finding 4-yearly centred moving averages:

Year t	Percentage of Enrolment y_t	4–yearly moving total	4–yearly moving average	2 unit moving total	4–yearly centred moving averages trend value
1960	0				
1965	3				
		10	2.5		
1970	3			6	3
		14	3.5		
1975	4			7.5	3.75

		16	4		
1980	4			8.75	4.375
		19	4.75		
1985	5			10.5	5.25
		23	5.75		
1990	6			12.5	6.25
		27	6.75		
1995	8			14.75	7.375
		32	8		
2000	8				
2005	10				4.5

Miscellaneous Exercise 4 | Q 4.1 | Page 69

Solve the following problem :

Fit a trend line to the data in Problem IV (1) by the method of least squares.

Solution: In the given problem, n = 9 (odd), middle t – value is 1978, h = 1

$$u = \frac{t - \text{middle value}}{h} = \frac{t - 1978}{1} = t - 1978$$

We obtain the following table.

Year t	Production y_t	u = t – 1978	u^2	uy_t	Trend Value
1974	0	–4	16	0	3.8001
1975	4	–3	9	–12	4.4334
1976	9	–2	4	–18	5.0667
1977	9	–1	1	–9	5.7
1978	8	0	0	0	6.3333
1979	5	1	1	5	6.9666
1980	4	2	4	8	7.5999
1981	8	3	9	24	8.2332
1982	10	4	16	40	8.8665
Total	**57**	**0**	**60**	**38**	

From the table, n = 9,

$$\sum y_t = 57, \sum u = 0, \sum u^2 = 60, \sum uy_t = 38$$

The two normal equations are:

$$\sum y_t = na' + b' \sum u \text{ and } \sum uy_t, = a' \sum u + b' \sum u^2$$

$\therefore$ 57 = 9a' + b'(0) ...(i) and

38 = a'(0) + b'(60) ...(ii)

From (i), $a' = \frac{57}{9} = 6.3333$

From (ii), $b' = \frac{38}{60} = 0.6333$

$\therefore$ The equation of the trend line is $y_t = a' + b' u$

i.e., $y_t = 6.3333 + 0.6333 u$, where $u = t - 1978$.

Miscellaneous Exercise 4 | Q 4.11 | Page 70

Solve the following problem :

Fit a trend line to data in Problem 10 by the method of least squares.

Solution: In the given problem, n = 8 (even), two middle t – values are 1980 and 1981, h – 1

$$u = \frac{\text{t - mean of two middle values}}{\frac{h}{2}} = \frac{t - 1980.5}{\frac{1}{2}} = 2(t - 1980.5)$$

We obtain the following table.

Year t	No. of boxes (in ten thousands) y_t	u = 2(t – 1980.5)	u^2	uy_t	Trend Value
1977	1	–7	49	–7	1.5836
1978	0	–5	25	0	2.5240

1979	3	–3	9	–9	3.4644
1980	8	–1	1	–8	4.4048
1981	10	1	1	10	5.3452
1982	4	3	9	12	6.2856
1983	5	5	25	25	7.2260
1984	8	7	49	56	8.1664
Total	**39**	**0**	**168**	**79**	

From the table, n = 8,

$$\sum y_t = 39, \sum u = 0, \sum u^2 = 168, \sum uy_t = 79$$

The two normal equations are:

$$\sum y_t = \mathrm{na}\prime + \mathrm{b}\prime \sum u \text{ and } \sum uy_t, = a\prime \sum u + b\prime \sum u^2$$

$\therefore$ 39 = 8a' + b'(0) ...(i) and

79 = a'(0) + b'(168) ...(ii)

From (i), $a' = \frac{39}{8} = 4.875$

From (ii), $b' = \frac{79}{168} = 0.4702$

$\therefore$ The equation of the trend line is $y_t = a' + b'u$

i.e., $y_t = 4.875 + 0.4702\ u$, where $u = 2(t - 1980.5)$.

Miscellaneous Exercise 4 | Q 4.12 | Page 70

Solve the following problem :

Obtain trend values for data in Problem 10 using 3-yearly moving averages.

Solution: Construct the following table for finding obtaining 3-yearly moving averages.

Year t	Production (in ten thousands) y_t	3–moving total	3–yearly moving averages ten value
1997	1	–	–
1978	0	4	1.3333
1979	3	11	3.6667
1980	8	21	7
1981	10	22	7.3333
1982	4	19	6.3333
1983	5	17	5.6667
1984	8	–	–

Miscellaneous Exercise 4 | Q 4.13 | Page 70

Solve the following problem :

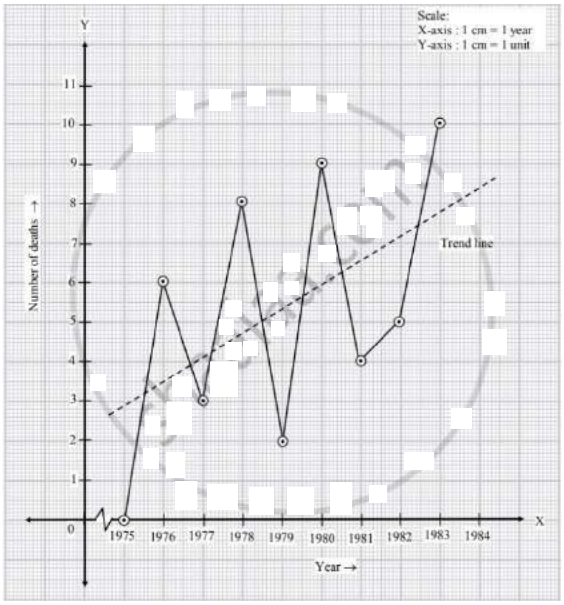
Following table shows the number of traffic fatalities (in a state) resulting from drunken driving for years 1975 to 1983.

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983
No. of deaths	0	6	3	8	2	9	4	5	10

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and number of deaths on Y-axis, we plot the points for number of deaths corresponding to year. Joining these points we get the graph of time

series, we fit the trend line as shown in the following graph.



Miscellaneous Exercise 4 | Q 4.14 | Page 70

Solve the following problem :

Fit a trend line to data in Problem 13 by the method of least squares.

Solution: In the given problem, n = 9 (odd), middle t – value is 1979, h – 1

$$u = \frac{t - \text{middle value}}{h} = \frac{t - 1979}{1} = t - 1979$$

We obtain the following table.

Year t	No. of deaths y_t	u = t – 1979	u^2	uy_t	Trend Value
1975	0	–4	16	0	2.5554
1976	6	–3	9	–18	3.2221
1977	3	–2	4	–6	3.8888
1978	8	–1	1	–8	4.5555
1979	2	0	0	0	5.2222
1980	9	1	1	9	5.8887
1981	4	2	4	8	6.5556
1982	5	3	9	15	7.2223
1983	10	4	16	40	7.8890
Total	**47**	**0**	**60**	**40**	

From the table, n = 9,

$$\sum y_t = 47, \sum u = 0, \sum u^2 = 60, \sum uy_t = 40$$

The two normal equations are:

$$\sum y_t = na' + b' \sum u \text{ and } \sum uy_t, = a' \sum u + b' \sum u^2$$

$\therefore$ 47 = 9a' + b'(0) ...(i) and

40 = a'(0) + b'(60) ...(ii)

From (i), $a' = \frac{47}{9} = 5.2222$

From (ii), $b' = \frac{40}{60} = 0.6667$

$\therefore$ The equation of the trend line is $y_t = a' + b'u$

i.e., $y_t = 5.2222 + 0.6667\ u$, where $u = t - 1979$.

Miscellaneous Exercise 4 | Q 4.15 | Page 70

Solve the following problem :

Obtain trend values for data in Problem 13 using 4-yearly moving averages.

Solution: Construct the following table for finding 4-yearly centred moving averages.

Year t	No. of deaths y_t	4–yearly moving average	4–yearly moving Averages	2 unit moving total	4–yearly centred moving averages trend value
1975	0				
1976	6				
		17	4.25		
1977	3			9	4.5
		19	4.75		
1978	8			10.25	5.125

		22	5.5		
1979	2			11.25	5.625
		23	5.75		
19880	9			1075	5.375
		20	5		
1981	4			12	6
		28	7		
1982	5				
1983	10				

Miscellaneous Exercise 4 | Q 4.16 | Page 70

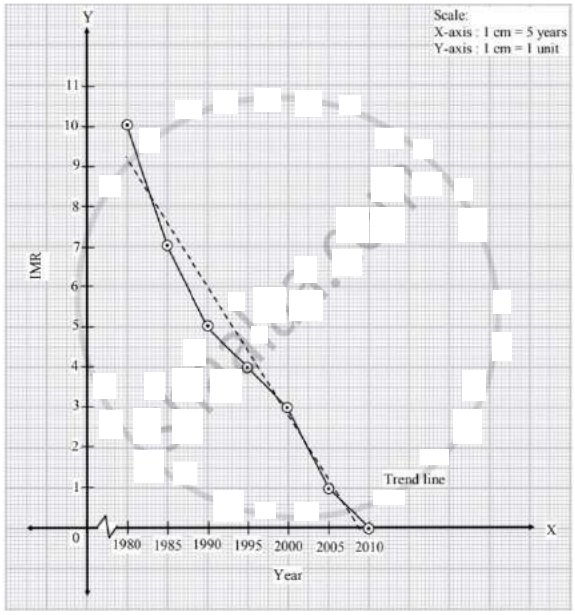
Solve the following problem :

Following table shows the all India infant mortality rates (per '000) for years 1980 to 2010.

Year	1980	1985	1990	1995	2000	2005	2010
IMR	10	7	5	4	3	1	0

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and mortality rate on Y-axis, we plot the points for mortality rate corresponding to years. Joining these points we get the graph of time series. We fit the trend line as shown in the following graph.



Miscellaneous Exercise 4 | Q 4.17 | Page 70

Solve the following problem :

Fit a trend line to data in Problem 16 by the method of least squares.

Solution: In the given problem, n = 7 (odd), middle t – value is 1995, h = 5

$$u = \frac{t - \text{middle value}}{h} = \frac{t - 1995}{1}$$

We obtain the following table.

Year t	Infant mortality rate y_t	$u = \frac{t - 1995}{5}$	u^2	uy_t	Trend Value
1980	10	–3	9	–30	8.9999
1985	7	–2	4	–14	7.4285
1990	5	–1	1	–5	5.8571
1995	4	0	0	0	4.2857
2000	3	1	1	3	2.7143
2005	1	2	4	2	1.1429
2010	0	3	9	0	–0.4285
Total	**30**	**0**	**28**	**–44**	

From the table, n = 7,

$$\sum y_t = 10, \sum u = 0, \sum u^2 = 28, \sum uy_t = -44$$

The two normal equations are:

$$\sum y_t = \text{na}\prime + \text{b}\prime \sum u \text{ and } \sum uy_t, = a\prime \sum u + b\prime \sum u^2$$

$\therefore$ 30 = 7a' + b'(0) ...(i) and

– 44 = a'(0) + b'(28) ...(ii)

From (i), a' = $\frac{30}{7}$ = 4.2857

From (ii), b' = $\frac{-44}{28}$ = 1.5714

$\therefore$ The equation of the trend line is y_t = a' + b'u

i.e., y_t = 4.2857 – 1.5714 u, where u = $\frac{t - 1995}{5}$.

Miscellaneous Exercise 4 | Q 4.18 | Page 70

Solve the following problem :

Obtain trend values for data in Problem 16 using 3-yearly moving averages.

Solution: Construct the following table for finding 3-yearly moving averages.

Year t	Infant mortality rate y_t	3–yearly moving total	3–yearly moving averages trend value
1980	10	–	–
1985	7	22	7.3333
1990	5	16	5.3333
1995	4	12	4
2000	3	8	2.6667
2005	1	4	1.3333
2010	0	–	–

Miscellaneous Exercise 4 | Q 4.19 | Page 70

Solve the following problem :

Following tables shows the wheat yield (‘000 tonnes) in India for years 1959 to 1968.

Year	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968
Yield	0	1	2	3	1	0	4	1	2	10

Fit a trend line to the above data by the method of least squares.

Solution: In the given problem, n = 10 (even), two middle t – value are 1963 and 1964, h = 1

$$u = \frac{t - \text{mean of two middle values}}{\frac{h}{2}} = \frac{t - 1963.5}{\frac{1}{2}} = 2(t - 1963.5)$$

We obtain the following table.

Year t	**Yield (in '000 tonnes) y_t**	**u = 2(t – 1963.5)**	**u^2**	**uy_t**	**Trend Value**
1959	0	–9	81	0	–0.1632
1960	1	–7	49	–7	0.4064
1961	2	–5	25	–10	0.9760
196	3	–3	9	–9	1.5456
1963	1	–1	1	–1	2.1152
1964	0	1	1	0	2.6848
1965	4	3	9	12	3.2544
1966	1	5	25	5	3.8240
1967	2	7	49	14	4.3936
1968	10	9	81	90	4.9632
Total	**24**	**0**	**330**	**94**	

From the table, n = 10,

$$\sum y_t = 24, \sum u = 0, \sum u^2 = 330, \sum uy_t = 94$$

The two normal equations are:

$$\sum y_t = \text{na}\prime + \text{b}\prime \sum u \text{ and } \sum uy_t, = a\prime \sum u + b\prime \sum u^2$$

$\therefore$ 24 = 10a' + b'(0) ...(i) and

94 = a'(0) + b'(330) ...(ii)

From (i), a' = $\frac{24}{10}$ = 2.4

From (ii), b' = $\frac{94}{330}$ = 0.2848

$\therefore$ The equation of the trend line is y_t = a' + b'u

i.e., y_t = 2.4 + 0.2848 u, where u = 2(t – 1963.5).

Miscellaneous Exercise 4 | Q 4.2 | Page 69

Solve the following problem :

Following data shows the number of boxes of cereal sold in years 1977 to 1984.

Year	1977	1978	1979	1980	1981	1982	1983	1984
No. of boxes in ten thousand	1	0	3	8	10	4	5	8

Fit a trend line to the above data by graphical method.

Solution: Taking year on X-axis and number of boxes on Y-axis, we plot the points for number of boxes corresponding to years. Joining these points, we get the graph of the

time series. We fit the trend line as shown in the following graph.